School of Information Technology
Indian Institute of Technology, Kharagpur

IT 60108: Soft Computing Applications
Mid-Semester Examination
Spring, 2015-2016

SAMPLE ANSWERS

1. Which of the following is/ are fuzzy set(s). Justify your answer **[4 x 2=8]**

(a) $A = \{(x_1, 0), (x_2, 0), (x_3, 0), (x_4, 0)\}$ defined over a universe of discourse $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$

Answer:

A is a fuzzy set defined over X, the universe of discourse.

(b) $B = \left\{(x, \mu_B(x)) \middle| x \in Z, set\ of\ all\ integers\ and\ \mu_B(x) = \frac{1-x}{1+x}\right\}$

Answer:

B is a **not** a fuzzy set because :

(i) $\mu_B(x)$ is undefined for $x = -1$.

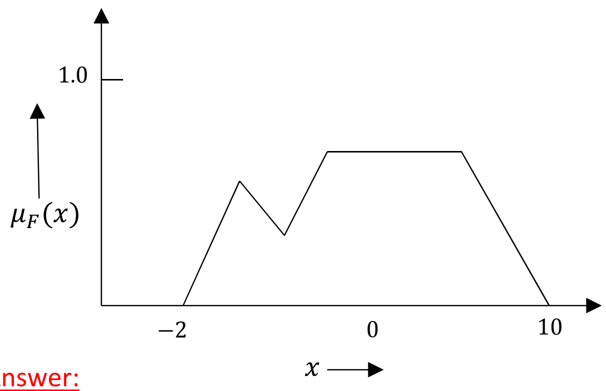
(ii) $\mu_B(x)$ is negative for same value $x \in Z$.

(c) $C = D \times E$ where D and E are two fuzzy sets and $\times$ denotes the Cartesian product of two fuzzy sets.

Answer:

C is **not** a fuzzy set as the Cartesian product of two fuzzy sets is a fuzzy relation.

(d)



Answer:

The graph shows a graphical representation of a fuzzy set F defined over the universe of discourse, $X = [-2, 10]$

2. Find the results of the fuzzy operations as instructed in the following: **[2 x 4=8]**

(a) $R = A \times B$ where

$$A = \left\{\frac{0.1}{x_1}, \frac{0.2}{x_3}, \frac{0.5}{x_5}\right\}$$

$$B = \left\{\frac{0.6}{x_2}, \frac{0.8}{x_3}, \frac{1.0}{x_6}\right\}$$

Answer:

A and B can be re-written as:

$$A = \left\{\frac{0.1}{x_1}, \frac{0.0}{x_2}, \frac{0.2}{x_3}, \frac{0.5}{x_5}, \frac{0.0}{x_6}\right\}$$

$$B = \left\{\frac{0.0}{x_1}, \frac{0.6}{x_2}, \frac{0.8}{x_3}, \frac{0.0}{x_5}, \frac{1.0}{x_6}\right\}$$

Now, $A \times B =$

X	x_1	x_2	x_3	x_5	x_6
x_1	0.0	0.1	0.1	0.0	0.1
x_2	0.0	0.0	0.0	0.0	0.0
x_3	0.0	0.2	0.2	0.0	0.2
x_5	0.0	0.5	0.5	0.0	0.5
x_6	0.0	0.0	0.0	0.0	0.0

(b) λ cut of the implication, $If\ x\ is\ A\ then\ y\ is\ B$, where

$A = \{(x_1, 0.1), (x_2, 0.3), (x_3, 0.2)\}$

$B = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.1)\}$

and both are defined over $X = \{x_1, x_2, x_3, x_4\}$ and $\lambda = 0.7$

Answer:

Using Zadeh's Max-min rule of implication, $If\ x\ is\ A\ then\ y\ is\ B$ can be represented as

$R = (A \times B) \cup (\bar{A} \times X$

Now, we have $A \times B =$

X	x_1	x_2	x_3
x_1	0.1	0.1	0.1
x_2	0.2	0.3	0.1
x_3	0.2	0.2	0.1

$\bar{A} \times X =$

X	x_1	x_2	x_3
x_1	0.9	0.9	0.9
x_2	0.7	0.7	0.7
x_3	0.8	0.8	0.8

Hence, $R = (A \times B) \cup (\bar{A} \times X)$=

X	x_1	x_2	x_3
x_1	0.9	0.9	0.9
x_2	0.7	0.7	0.7
x_3	0.8	0.8	0.8

Finally, $R_{\lambda=0.7} =$

X	x_1	x_2	x_3
x_1	0.0	0.0	0.0
x_2	0.7	0.7	0.7
x_3	0.0	0.0	0.0

3. Any road is characterized with two fuzzy linguistics WIDE and NARROW whereas a journey is characterized with two fuzzy linguistics HIGH RISK and LOW RISK. The universe of discourses of road and journey are $\{Large, Medium, Small\}$ and $\{High, Moderate, Low\}$ respectively.

   A road and journey are associated with fuzzy implication, $If\ road\ is\ WIDE\ then\ driving\ is\ RISKY$. For the MG Road, it is given that

   $$Road\ is\ WIDE = \left\{\frac{0.3}{Large}, \frac{0.5}{Medium}, \frac{0.7}{Small}\right\}$$

   $$Road\ is\ RISKY = \left\{\frac{0.9}{High}, \frac{0.7}{Moderate}, \frac{0.6}{Low}\right\}$$

   $$Driving\ on\ M.G.\ road\ is\ RISKY = \left\{\frac{0.7}{High}, \frac{0.6}{Moderate}, \frac{0.5}{Low}\right\}$$

   What is the fuzzy set that, $.G.\ road\ is\ NARROW$? **[10]**

   Answer:

   To solve the above problem, we can apply "Generalized Modus Tollen (GMT)" rule which is as follows:

   $If\ x\ is\ A\ then\ y\ is\ B$

   $y\ is\ B'$ _ _ _ _ _ _ _

   $x\ is\ A'$

   Using Max-min composition rule, A'can be calculated as $A' = B' \circ R(x, y)$

   where $R(x, y) = (A \times B) \cup (\bar{A} \times Y)$

   For the given problem:

   $$A: M.G. Road\ is\ WIDE \equiv \left\{\frac{0.3}{Large}, \frac{0.5}{Medium}, \frac{0.7}{Small}\right\}$$

   $$B: Journey\ M.G. Road\ is\ RISKY \equiv \left\{\frac{0.9}{High}, \frac{0.7}{Moderate}, \frac{0.6}{Low}\right\}$$

   Hence, $R(x, y) = (A \times B) \cup (\bar{A} \times Y)$

$A \times B =$

	H	M	L
L	0.3	0.3	0.3
M	0.5	0.5	0.5
S	0.7	0.7	0.6

$\bar{A} \times Y =$

	H	M	L
L	0.7	0.7	0.7
M	0.5	0.5	0.5
S	0.3	0.3	0.3

$R(x, y) =$

		RISKY		
		H	M	L
ROAD	L	0.7	0.7	0.7
	M	0.5	0.5	0.5
	S	0.7	0.7	0.6

$A': Driving\ on\ M.G.Road\ is\ RISKY \equiv \left\{\frac{0.7}{High}, \frac{0.6}{Moderate}, \frac{0.5}{Low}\right\}$

Therefore,

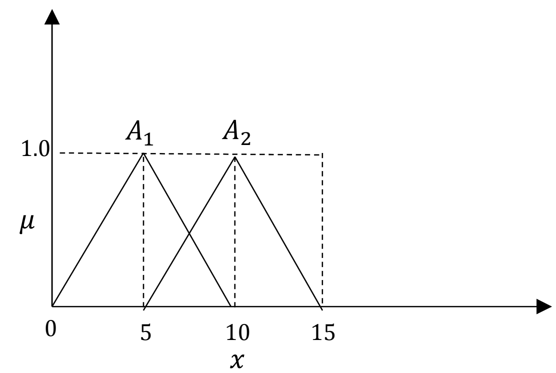
$B': M.G.Road\ is\ NARROW\ is\ B' = A' \circ R(x, y) \equiv \left\{\frac{0.5}{Large}, \frac{0.5}{Medium}, \frac{0.7}{Small}\right\}$

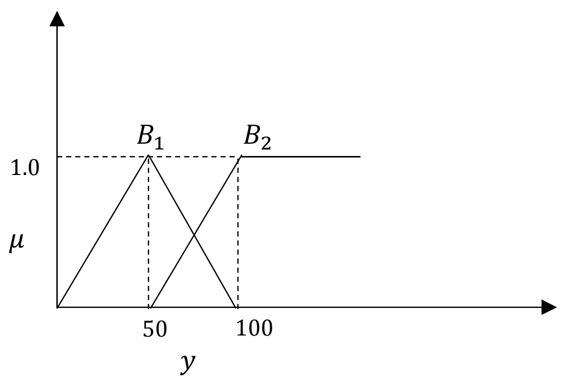
4. In a fuzzy controller for two input $x = 6$ and $y = 25$, two fuzzy rules are fired as below:

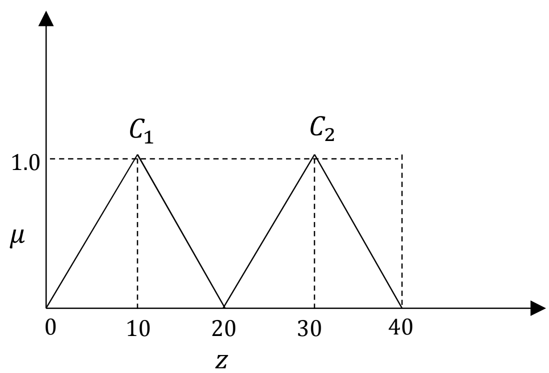
   $R_i: IF\ x\ is\ A_1\ AND\ y\ is\ B_1\ THEN\ z\ is\ C_1$

   $R_j: IF\ x\ is\ A_2\ AND\ y\ is\ B_2\ THEN\ z\ is\ C_2$

   The fuzzy sets involved in R_i and R_j are known as given below:





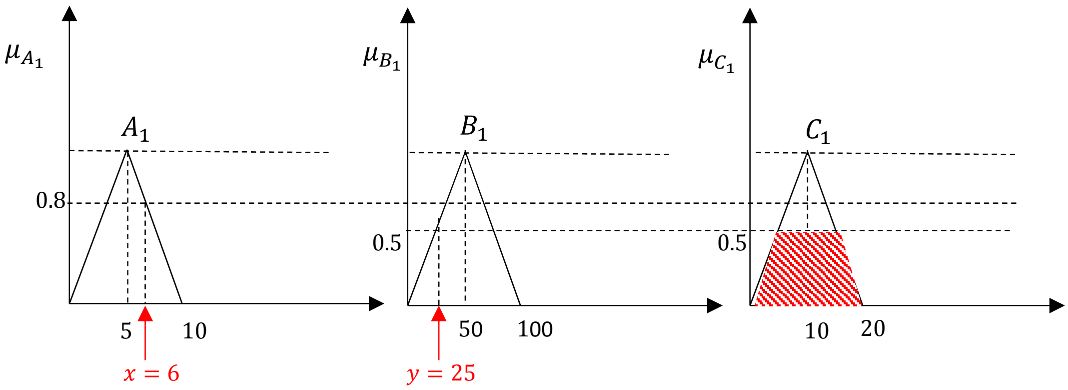


(a) Graphically show the combined output due to R_i and R_j for $x = 6$ and $y = 25$.

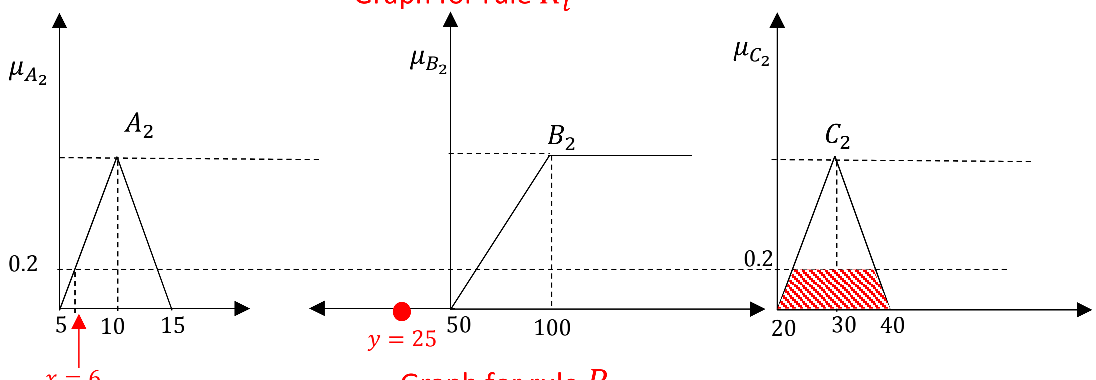
Answer:

To show the fuzzy output graphically

The graphs for the rules R_i and R_j are shown below:

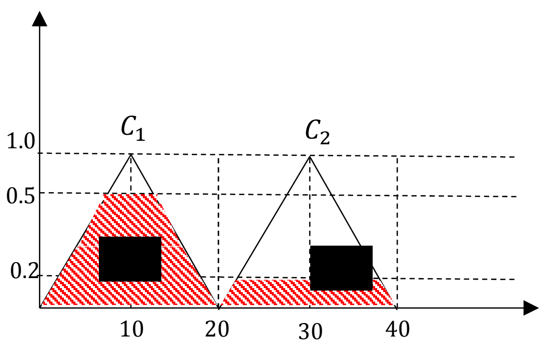


Graph for rule R_i

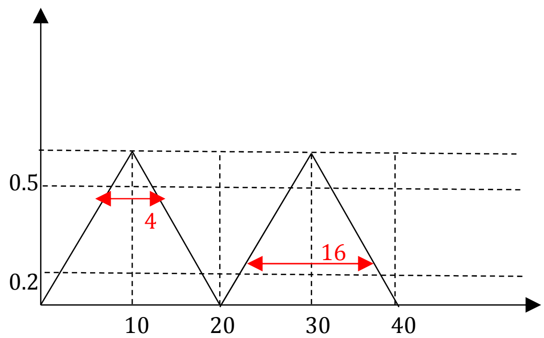


Graph for rule R_j

Thus, combined fuzzy output is as highlighted in the graph with red,



(b) Apply COS (Center of Sum) defuzzification method to obtain the crisp value of the output when $x = 6$ and $y = 25$. **[6 x 2=12]**



$A_1 = \frac{1}{2}(4 + 20)(0.5)$, centre $x_1 = 10$

$A_2 = \frac{1}{2}(16 + 20)(0.2)$, centre $x_2 = 30$

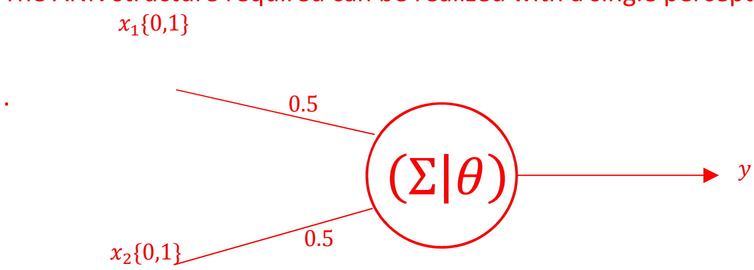
$\text{Output} = \frac{60 + 108}{6.0 + 3.6} = \frac{168}{9.6} = 17.5$

5. Write brief answers to the following questions: **[4 x 4=16]**

(a) Draw an ANN with the minimum number of perceptron which would classify input pattern $00, 01, 10$ and 11 into two classes 0 and 1 following OR-logic.

Answer:

The ANN structure required can be realized with a single perceptron as shown below:



Here, $I = 0.5(x_1) + 0.5(x_2)$ and transfer function with threshold limit $\theta = 0.5$ is as follows:

$If\ I \geq \theta\ then\ y = 1\ ELSE\ y = 0$

Note: Value of θ, w_1 and w_2 are not necessarily unique.

(b) The $Tanh - sigmoid$ transfer function ∅ is defined as follows:

$$\emptyset(I) = \frac{e^{\theta i} - e^{-\theta i}}{e^{\theta i} + e^{-\theta i}}$$

where symbols bear usual meaning.

Prove that $\frac{\partial \emptyset}{\partial I} = \theta(1 + \emptyset(I))(1 - \emptyset(I))$

Answer:

$$\emptyset(I) = \frac{e^{\theta i} - e^{-\theta i}}{e^{\theta i} + e^{-\theta i}} = \frac{u}{v}(say)$$

$$\therefore \ \frac{\partial \emptyset}{\partial I} = \frac{v.\frac{\partial u}{\partial I} - u.\frac{\partial v}{\partial I}}{v^2}$$

$$= \theta.\frac{(e^{\theta i} + e^{-\theta i})^2 - (e^{\theta i} - e^{-\theta i})^2}{(e^{\theta i} + e^{-\theta i})^2}$$

$$= \theta.\left[1 - (\frac{e^{\theta i} - e^{-\theta i}}{e^{\theta i} + e^{-\theta i}})^2\right]$$

$$= \ \theta.[1 - \{\emptyset(I)\}^2]$$

$$= \theta(1 + \emptyset(I))(1 - \emptyset(I)),$$

Hence proved.

(c) State the Delta rule, which is usually followed in Back propagation algorithm. Is the rule applicable to any type of ANN?

Answer:

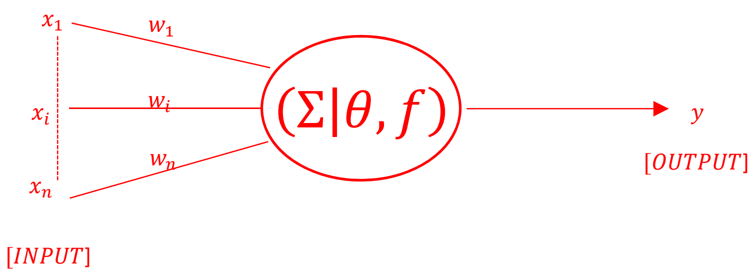
If E denotes error of an ANN and θ is an ANN parameter, then "Delta rule" is defined as

$$\Delta\theta \ = \text{ŋ}.\frac{\partial E}{\partial \theta}$$

where ŋ is a constant and is known as "learning rate". Such a Delat rule is applicable to all ANNs.

(d) Draw a symbolic diagram (also called bubble) of a perceptron and clearly show the different unknown parameters in it.

Answer:



Unknown parameters in the perceptron are:

$W = [w_1, w_2, \dots \dots \dots \dots., w_n]$

$f = Transfer\ function.$

$\theta = Threshold\ limit\ in\ the\ transfer\ function\ f.$

6. Answer the following: **[3x 8=24]**

(a) Draw a MLFFNN having $l - m - n$ configuration. Clearly show its network parameters.

Answer:

Refer Slides-10 to 13 of ***Artificial Neural Network: Architectures***.

(b) Suppose, $T =< T_o, T_I >$ is a training data for the supervised learning of a $l - m - n$ network. If $I_i \in T_I$ is the i^{th} input applied to the network, then express the error at the k^{th} perceptron in the output layer. Also, obtain the expression for total error E due to all $I_i \in T_I$.

Answer:

Let T_{O_K} denote the actual output and O_{O_K} denote the observed output due to $I_i \in T_I$.

The error at the k^{th} neuron in the output layer (using mean square error) is

$$e_K = \frac{1}{2}(T_{O_K} - O_{O_K})^2$$

If there are n neutrons at the output layer, then error considering all output neurons due to $I_i \in T_I$ is

$$e = \sum_{K=1}^{n} e_K = \frac{1}{2}\sum_{K=1}^{n}(T_{O_K} - O_{O_K})^2$$

Thus, total error due to all input $I_i \in T_I$ will be obtained as

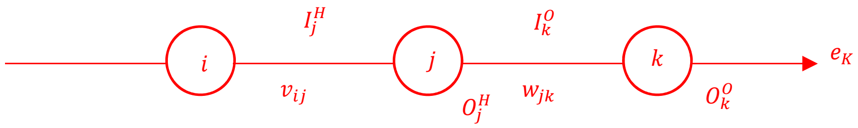
$$E = \sum_{\forall I_i \in T_I} e = \frac{1}{2}\sum_{\forall t \in T}\sum_{K=1}^{n}(T_{O_K} - O_{O_K})^2$$

(c) If w_{ij} and v_{jk} denotes the weight of the link from an i^{th}neuron in the input layer to the j^{th} neuron in the hidden layer to the k^{th} neuron in the output layer and e_K denotes the error of the k^{th} neuron on in the output layer, then write down the chain rule of differentiation to calculate the following:

$\frac{\partial e_K}{\partial w_{ij}}$ and $\frac{\partial e_K}{\partial v_{ij}}$

Clearly mention all the symbols used in the rule you have stated.

Answer:



Here,

I_j^H = input to the j^{th} neuron at the hidden layer.

O_j^H = output from the j^{th} neuron at the hidden layer.

I_k^O = input to the k^{th} neuron at the output layer.

O_k^O = observed output of the k^{th} neuron at the output layer.

Then, the chain rule of differentiation can be expressed as

$$\frac{\partial e_K}{\partial w_{jK}} = \frac{\partial e_K}{\partial O_K^O} \cdot \frac{\partial O_K^O}{\partial I_K^O} \cdot \frac{\partial I_K^O}{\partial w_{jK}}$$

and,

$$\frac{\partial e_K}{\partial v_{ij}} = \frac{\partial e_K}{\partial O_K^O} \cdot \frac{\partial O_K^O}{\partial I_K^O} \cdot \frac{\partial I_K^O}{\partial O_j^H} \cdot \frac{\partial O_j^H}{\partial I_j^H} \cdot \frac{\partial I_j^H}{\partial v_{ij}}$$